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A unified approximation framework for parameter estimation in general linear PDE models has been completed. This framework has provided the theoretical basis for a number of identification problems on which these investigators have made significant progress. These include: (i) nondestructive evaluation techniques of composite materials using thermal probes, (ii) estimation of damping in composite material beams from vibration experiments. In connection with item (ii) it has been shown conclusively that an identification of damping mechanisms in the partial differential equation of a composite beam cannot be accomplished by the use of experimental modal analysis. This is a major result in the theory of identifying damping mechanisms in flexible structures. The group has also studied questions related to the determination of irregularities (corrosions, cracks, delaminations, etc.) in composite materials using boundary observations of temperatures after known heat fluxes have beeen applied to the boundary. Successful efforts using experimental data with the theoretical and computational ideas developed by this group are reported. Substantial progress has been made on the development of a statistical framework (including hypothesis testing algorithms) to use in comparing the suitability of PDE models in least squares fits to data. A number of results on feedback stabilization of distributed parameter models have been obtained.

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Report prepared by:
H.T. Banks

INTRODUCTION

Funding in this grant was used to support in part a number of students, postdoctorals and long term visitors to the Center for Control Sciences in addition to support of the principal investigators on the contract: H.T. Banks, D.J. Inman, K. Ito, W.H. Fleming, C. Dafermos, and D. Gottlieb.

Postdoctoral long term visitors with support during the third year included K. Kunisch, B. Fitzpatrick, H. Tran, R.H. Fabiano, D.E. Woodward, H. Cudney, G. Barles, H. Talezer, and X. Geng. Students supported include D. Rebnord, G. Wade, M. Desai, K. Black, F. Fakhroo, X. Jiang, and Y. Wang.

This final report consists of a brief overview of the research for the three year period September 15, 1986 - September 15, 1989 followed by detailed summaries written by each of the investigators.

BRIEF OVERVIEW - SUMMARY

Banks and Ito have completed a unified approximation framework for parameter estimation in general linear PDE models. This framework has provided the theoretical basis for a number of identification problems on which we have made significant progress during the past several years. These include: (i) nondestructive evaluation techniques for composite materials using thermal probes, (ii) estimation of damping in composite material beams from vibration experiments.

In joint efforts with W. Winfree (NASA Langley Research Center) and F. Kojima (ICASE), Banks has studied questions related to determination of irregularities (corrosions, cracks, delaminations, etc.) in composite materials using boundary observations of temperatures after known heat fluxes have been applied to the boundary. Successful efforts using experimental data with the theoretical and computational ideas developed are continuing.

Significant progress has been made in our efforts to understand the difference between various types of internal damping in composite material beams. Using data from the MSL at Buffalo and in collaboration with D.J. Inman and H.H. Cudney, Jr., Banks, in joint efforts with Y. Wang and R.H. Fabiano, has developed computational techniques for estimation of stiffness, damping parameters and other structural parameters in beams. These techniques have been used with experimental data to study several types of damping including Kelvin-Voigt, spatial hysteresis, time hysteresis, and bending rate damping. Work in this area has been pursued with data taken in experiments carried out at the Air Force Astronautics Laboratory (AFAL) in a thermal vacuum chamber. Both viscous damping and the dependency of damping coefficients on temperature are being investigated.

Related efforts on damping in a 2D-grid structure using experimental data from AFAL were pursued by Banks and a graduate student, D.A. Rebnord.

Banks, in joint efforts with Rosen and Reich has begun development of a framework (similar to that for linear systems developed by Banks and Ito), that can be applied to nonlinear PDE systems of first and second order in time. Efforts to use these theoretical and computational ideas with experimental data for thermal/viscoelastic coupled systems and nonlinear damping models are planned.

Banks and Fitzpatrick (a May, 1988 PhD. graduate and 1989-90 postdoc in CCS) have made substantial progress on development of a statistical framework (including hypothesis testing algorithms) to use in comparing the suitability of models in least squares fits to data. The theoretical basis involves establishing asymptotic (as sample size becomes infinite) limiting distributions for ratios of residual sums of squares obtained in the model fitting. The resulting techniques have proved most useful in the damping studies reported on above.

Banks and C. Wang (also a 1988 PhD. graduate, now at USC) have developed approximation techniques for feedback controls in nonautonomous parabolic systems. These ideas, developed in the context of general evolution systems, can be applied to general tracking problems for hyperbolic systems with boundary damping. Such systems have been studied by Banks, Keeling, Propst, and Silcox in connection with noise suppression by active feedback in aircraft. A theory for feedback control is being developed and efforts on computational techniques to be used with data from Silcox's lab at NASA

Langley are being pursued.

The Center's laboratory facilities, housed in the MSL at SUNYAB, have been expanded to allow slewing control experiments as well as improved parameter identification experiments. In addition to the new hardware capabilities added to the laboratory, experiments have begun at the AFAL at Edwards Air Force Base in one of the thermo-vac facilities in conjunction with Dr. Alok Das. The laboratory experiments have motivated a number of numerical and theoretical results including work in model modification via eigenstructure assignment, conclusive evidence that modal analysis fails to describe damping properly, control structure interaction results, and some results on finite dimensional control theory.

The MSL facility was improved by the addition of a new accelerometer prototype which allows the simultaneous measurements of linear and rotational acceleration at the same point on a structure. This represents the first laboratory capability to identify coefficients in partial differential equations representing both bending and torsional vibrations. Previously, members of the applied mathematics and mechanics communities have been forced to treat these effects in a uncoupled fashion. This new device will allow the parameter identification community to consider a more complete set of partial differential equations to describe the motion of flexible structures.

Members of the Center (H.T. Banks, E. Garcia, D.J. Inman) visited the Air Force Astronautics Laboratory (E. Garcia stayed for a period of two weeks) on numerous occasions to initiate a series of vibration tests on a variety of materials. A standard size beam was chosen for the research. The samples of this beam were made of three different materials (steel, graphite expoxy and antiphon) with three very different damping properties. Tests of the steel beam were performed at different pressures and temperatures in order to study the effects of temperature differences and lack of air resistance on damping in flexible structures. These two effects have not been adequately identified previously and are essential for understanding the behavior of flexible structures in orbit.

A long standing problem in the analysis of structures using finite element modeling (FEM) has been the inability of standard modal models obtained from modal analysis experiments to compare well with an FEM of the same structure. This problem has been solved in a non-physical way by applying eigenstructure assignment techniques used to match experimental data to the analytical FEM. The assigned eigenstructure is taken to be the experimentally determined eigenvalues and eigenvectors. The resulting gain matrices are used as corrections to the FEM.

It has been shown quite conclusively that an identification of damping mechanisms in the partial differential equation of a composite beam cannot be accomplished by the use of experimental modal analysis. This is a major result in the theory of identifying damping mechanisms in flexible structures and shows explicitly why a partial differential equation description is necessary to determine physically based damping models in flexible structures. Details can be found in papers by Banks and Inman and by Cudney and Inman cited elsewhere in this report.

The nature of the interaction between a structure, a control law, and the actuator used to implement the control law on a system consisting of a cantilevered composite beam and a proof-mass actuator was considered. This interaction was found to be potentially destabilizing with the choice of actuator break frequency relative to the lowest structure frequency being critical. These results are delineated in a paper by Zimmerman and Inman.

Motivated by models for linear viscoelastic beams, Ito and Fabiano have studied the integro-differential equation $\ddot{u}(t) + A(u(t) + \int_{-r}^{0} a(\theta)u(t+\theta)d\theta) = f(t)$ on a Hilbert space H where A is a positive self-adjoint operator on H. They have developed results for both strongly and weakly singular kernel a. Well-posedness and the exponental stability of solutions were established using linear semigroup theory. They also showed that under certain conditions on a, the associated solution semigroup is differentiable. This implies that the damping rate is (at least) proportional to log | the natural frequency |. They developed an approximation framework for numerical schemes for the above equation.

Banks and Ito have studied a class of boundary control problems for parabolic systems. The study also focused on strongly damped second-order evolution equations with boundary controls, which include the Euler-Bernoulli beam model with Kelvin-Voigt damping. They developed an approximation theory for linear quadratic regulator (LQR) problems for such systems based on the sesquilinear form framework. This not only extends an existing theory for the bounded input-output operator case but also of provides a stronger convergence result.

Motivated by this study, Ito and Tran developed a general approximation framework for LQR problems for infinite dimensional linear systems involving unbounded input-output operators. An objective of their study is to construct a stabilizing optimal feedback control law for systems with delays in the control.

Often, one formulates parameter estimation problems as constrained optimization problems in Hilbert spaces. They are constrained because a class of admissible parameters q may be characterized by $|q|_Q \leq \gamma$ for $|\cdot|_Q$ a chosen semi-norm of the parameter space Q. The performance index may include the regularization therm $\frac{\beta}{2}|q|_Q^2$ in order to obtain a stable generalized inverse map. A question, here, is how to choose a optimal parameter β (or γ) for such problems. Ito and Kunisch have begun to study this problem, analyzing the sensitivity of solutions with respect to parameters involved in optimization (e.g., β or γ), and trying to construct a procedure to find an optimal regularization constant β (or γ).

RESEARCH SUMMARIES

Research Summary: H.T. Banks

During the past several years, we have been engaged in efforts which are primarily aimed at the understanding of damping mechanisms and models in distributed parameter systems for flexible structures. Our efforts entail several specific components.

I. Two dimensional grid-like structures: We have considered "plates with large, periodic holes" as a precursor to the study of 3-D grids and trusses that are commonly encountered in large space structures. Our efforts on damping in 2-D grids were motivated in part by and in collaboration with Dr. Alok Das and the group at the USAF Astronautics Lab (a part of the experimental facilities for the NASA/DOD CSI technology research initiative). We have considered several modeling approaches to the problems.

A: Classical Love-Kirchoff plate theory: We considered a "plate with holes" with external viscous damping and Kelvin-Voigt internal damping. The system equations are defined on a perforated domain $\Omega_{\varepsilon,\mu}$ which is a rectangular plate with numerous, repeated periodically, rectangular holes. Here μ and ε are parameters related to the size of the grid members and the size and frequency of the holes, respectively. On this domain we have the Love -Kirchoff equations with damping:

$$\rho h\frac{\partial^2 u}{\partial t^2}+\gamma\frac{\partial u}{\partial t}-\frac{\partial^2 M_x}{\partial x^2}-2\frac{\partial^2 M_{xy}}{\partial x\partial y}-\frac{\partial^2 M_y}{\partial y^2}=f(t,x,y), \tag{1}$$

$$(x,y)\in\Omega_{\varepsilon,\mu}\subset[0,1]\times[0,1]$$

$$\begin{aligned} u(0) &= u_0 \\ u_t(0) &= v_0 \end{aligned}$$

where

$$M_x=-D\frac{\partial^2 u}{\partial x^2}-\nu D\frac{\partial^2 u}{\partial y^2}-\frac{c_D I}{1-\nu^2}\frac{\partial^3 u}{\partial t\partial x^2}-\frac{\nu c_D I}{1-\nu^2}\frac{\partial^3 u}{\partial t\partial y^2}$$

(the moment due to bending transverse to an axis parallel to the x-axis),

$$M_y=-D\frac{\partial^2 u}{\partial y^2}-\nu D\frac{\partial^2 u}{\partial x^2}-\frac{c_D I}{1-\gamma^2}\frac{\partial^3 u}{\partial t\partial y^2}-\frac{\nu c_D I}{1-\gamma^2}\frac{\partial^3 u}{\partial t\partial x^2},$$

$$M_{xy}=-(1-\nu)D\frac{\partial^2 u}{\partial x\partial y}-\frac{c_D I}{1+\nu}\frac{\partial^3 u}{\partial x\partial y\partial t}.$$

Here h is the thickness of the plate, ρ is the mass density, $D=EI/(1-\nu^2)$ is the "stiffness" and ν is Poisson's ratio.

There are two types of boundary conditions that can be used with this system of equations: essential boundary conditions or those that must be directly imposed on solutions and test functions in any weak or variational formulation of system (1) and natural boundary conditions or those that need not be imposed in a weak or variational formulation of (1) since they are automatically satisfied by any weak solution that is sufficiently smooth. Combinations of several types of physical boundary conditions are important. For a clamped edge we have:

$$u = \frac{\partial u}{\partial x} = 0 \quad \text{on edges parallel to the } y\text{-axis},$$

$$u = \frac{\partial u}{\partial y} = 0 \quad \text{on edges parallel to the } x\text{-axis},$$

both of which are essential boundary conditions. For a free boundary (we use these for the "hole" edges plus for any exterior edge which is free) we have the natural boundary conditions

$$\left.\begin{array}{c} M_x = 0 \\ \frac{\partial M_x}{\partial x} + 2\frac{\partial M_{xy}}{\partial y} = 0 \end{array}\right. \text{on edges parallel to the } y\text{-axis},$$

$$\left.\begin{array}{c} M_y = 0 \\ \frac{\partial M_y}{\partial y} + 2\frac{\partial M_{xy}}{\partial x} = 0 \end{array}\right. \text{on edges parallel to the } x\text{-axis}.$$

Finally for simply supported edges we have

$$u = M_x = 0 \text{ on edges parallel to the } y\text{-axis},$$

$$u = M_y = 0 \text{ on edges parallel to the } x\text{-axis}.$$

We note that this boundary configuration requires one essential boundary condition ($u = 0$) and one natural boundary condition ($M_x = 0$ or $M_y = 0$).

A typical case study might combine several of these boundary conditions: e.g. a grid that is clamped on one side only, free on the other three sides plus the interior edges (the hole regions).

Among the parameters to "identified" from experiments are the damping parameters $q_0 = \gamma, q_2 = c_D I/(1-\nu^2), q_3 = \nu q_2$ and the stiffness parameters $q_1 = D, q_4 = \nu q_1$. We have developed a mathematical theory for approximation and parameter estimation in terms of a weak variational formulation of (1) wherein one seeks solutions $u(t) \in V \subset H^2(\Omega_{\epsilon,\mu})$ satisfying for all $\phi \in V$

$$\langle \rho h u_{tt}, \phi \rangle + \sigma_2(q)(u_t, \phi) + \sigma_1(q)(u, \phi) = \langle f, \phi \rangle,$$

where σ_1, σ_2 are appropriately defined "stiffness" and "damping" sesquilinear forms on functions defined over the domain $\Omega_{\epsilon,\mu}$. This theory is developed under a wide variety of combinations of boundary conditions, including all those that are physically important (details are given in [R], see Lemma 2.1). This convergence and stability theory is based on a nontrivial extension of the theoretical framework of Banks and Ito in [BI] and allows one to develop computational packages based on spline approximations (finite elements - cubic splines for the states, linear splines for the spatially dependent parameters). The resulting theory contains new and significant advances of a practical nature. For example, the earlier theory of [BI] required either displacement or velocity observations in the least squares parameter estimation algorithms. The new theory establishes that the I.D. schemes are theoretically sound if accelerometer data is used in the algorithms (this is true whenever the model involves damping of sufficient strength so that the damped system generates an analytic semigroup - e.g., as in the case with Kelvin-Voigt damping).

We have developed computational packages based on the above described approximation theory and used them to estimate damping (spatially varying) in examples of plates with holes. These are computationally difficult problems; even using vector machines, the computational domain $\Omega_{\epsilon,\mu}$ (a rectangular region with rectangular holes) presents a formidable challenge if one takes this direct approach. Based on these efforts, we conjecture that such a direct physical model approach would not prove feasible in dealing with the more complex truss structures that are the focus of our long term efforts.

B. Homogenization in plate models: One alternative to the direct modeling approach described above is the use of homogenization theory to develop "indirect" models for grids and trusses. We have made significant progress in our initial efforts in this direction durir~ the past year. The basic ideas involve a limiting procedure in which one approximates (through a precise but fairly complicated nonstandard mathematical double limit process - i.e., as $\epsilon, \mu \to 0$ in the domain) the direct physical model (the Love-Kirchoff model above) on a perforated (and computationally complicated) domain $\Omega_{\epsilon,\mu}$ by a "homogenized" system (this results in a more complex system if variable coefficients are involved in the original model) resulting in an indirect (non-physical parameters) model on a domain Ω which is equivalent to the original domain with the "holes" or perforations "filled in." That is, in examples for grid structures, the grid domain is replaced by a full plate; for truss structures, one obtains a solid multi-dimensional beam-like structure.

During the past year we have begun to develop both the mathematical theory and associated computational techniques to try to use these ideas in inverse problems for the estimation of physically meaningful damping and stiffness coefficients. Returning to the system (1) above, we outline some results for a "plate with holes" with clamped exterior boundaries. Then one seeks solutions $u_{\epsilon,\mu}$ to the system (1) lying in the state space

$$H_e^2(\Omega_{\epsilon,\mu}) = \{\phi \in H^2(\Omega_{\epsilon,\mu}) : \phi = \tfrac{\partial\phi}{\partial n} = 0 \quad \text{on the exterior boundaries of } \Omega_{\epsilon,\mu}\}.$$

This presents an extremely difficult computational problem which makes the inverse problem for damping and/or stiffness even more formidable (see the discussions above). Under the homogenization process, the limiting (homogenized) solution $w = \lim_{\epsilon,\mu\to 0} u_{\epsilon,\mu}$ (where the limit is taken in a weak* sense in $L^\infty(0,T;H_0^2(\Omega))$) satisfies the "homogenization system"

$$\begin{aligned} &\rho h\frac{\partial^2 w}{\partial t^2} + \gamma\frac{\partial w}{\partial t} + \frac{\partial^2}{\partial x^2}\{\frac{D(1-\nu^2)}{2}\frac{\partial^2 w}{\partial x^2} + \frac{c_D I}{2}\frac{\partial^3 w}{\partial t\partial x^2}\} \\ &+2\frac{\partial^2}{\partial x\partial y}\{D(1-\nu)\frac{\partial^2 w}{\partial x\partial y} + \frac{c_D I}{1+\nu}\frac{\partial^3 w}{\partial x\partial y\partial t}\} \\ &+\frac{\partial^2}{\partial y^2}\{\frac{D(1-\nu^2)}{2}\frac{\partial^2 w}{\partial y^2} + \frac{c_D I}{2}\frac{\partial^3 w}{\partial t\partial y^2}\} = f(t,x,y),(x,y)\in\Omega, \end{aligned} \tag{2}$$

with $w = \frac{\partial w}{\partial n} = 0$ on $\partial\Omega$ (i.e.,. $w \in H_0^2(\Omega)$).

We have developed computational packages to treat inverse problems for this system. Our simulation studies on the convergence of $u_{\epsilon,\mu}$ to w (and the associated "convergence" of estimated parameters in the respective inverse problems) are quite promising. We are currently investigating use of the model (2) with experimental data from the grid structure at USAF/AL.

II. Damping models in beams: We have continued our efforts involving the identification of damping mechanisms using experimental data with distributed parameter models for beams. Our investigations have focused on damping models in cantilevered Euler-Bernoulli beams with tip body. Our general model has the form

$$\rho\frac{\partial^2 u}{\partial t^2}+\gamma\frac{\partial u}{\partial t}+\frac{\partial^2}{\partial x^2}\{M_{INT}(t,x)\}=f(t,x), 0<x<\ell,$$

$$u(0,x)=\Phi(x) \tag{3}$$

$$u_t(0,x)=\Psi(x)$$

with boundary conditions:

(4)

clamped end $(x=0)$: $\quad u(t,0)=\frac{\partial u}{\partial x}(t,0)=0$

free end (with tip body) $(x=\ell)$:

(force balance) $[m_T\frac{\partial^2 u}{\partial t^2}-\frac{\partial}{\partial x}M_{INT}]_{x=\ell}=0$

(moment balance) $[J\frac{\partial^3 u}{\partial x\partial t^2}+M_{INT}]_{x=\ell}=0.$

Here M_{INT} represents the total internal moment (due to bending and damping), m_T is the mass of the tip body, J is its moment about its axis, and f is the applied external force.

Earlier efforts have concentrated on Kelvin-Voigt damping, time hysteresis damping, and spatial hysteresis. For example, in the case of spatial hysteresis the moment becomes

$$M_{INT}(t,x)=EI\frac{\partial^2 u}{\partial x^2}(t,x)+\int_x^\ell\int_0^\ell b(x,\xi)[\frac{\partial^2 u}{\partial t\partial x}(t,x)-\frac{\partial^2 u}{\partial t\partial x}(t,\xi)]d\xi$$

which reflects a nonlocal dependence of the damping on the rotational rate. This model has proved to be quite appropriate when modeling composite material beams (where longitudinal fibers can be expected to produce a nonlocal effect).

Our more recent investigations [BFW] have involved <u>bending rate</u> damping. In this model (which is popular with structural engineers) the damping is often assumed proportional to $\frac{\partial^3 u}{\partial t\partial x^2}$ and is often called <u>structural</u> or <u>square-root</u> damping. This latter terminology arises because <u>in special cases</u>, one finds $\frac{\partial^3 u}{\partial t \partial x^2} = A^{\frac{1}{2}} \frac{\partial u}{\partial t}$ where A is the stiffness operator ($Au = \frac{\partial^2}{\partial x^2}\{EI\frac{\partial^2 u}{\partial x^2}\}$ taken with appropriate boundary conditions).

However such a characterization does not hold for many practical situations of interest and, in particular, it does not hold for the case of cantilevered boundary conditions. But this damping can be investigated for cantilevered beams in the context of the general model (3), (4) based on direct physical principles (i.e., force and moment balance considerations) instead of abstract operator theory. An appropriate moment is given by

$$M_{INT}(t,x) = EI\frac{\partial^2 u}{\partial x^2}(t,x) + \eta\frac{\ell - x}{\ell}\int_0^x \frac{\partial^2 u}{\partial t \partial x}(t,\xi)d\xi + \eta\frac{x}{\ell}\int_x^\ell \frac{\partial^2 u}{\partial t \partial x}(t,\xi)d\xi$$

so that the damping term in (3) becomes - $\eta\frac{\partial^3 u}{\partial t \partial x^2}$. Based on this formulation, we developed the necessary mathematical and computational ideas (and resulting software) to investigate the use of bending rate damping in describing vibrations of composite material beams. As reported in [BFW], our efforts with experimental data suggest that this model is <u>not</u> particularly appropriate for composites. It is not as accurate in describing damping experiments as several other models (for example, both spatial hysteresis and time hysteresis models appear to be better models).

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Research Summary: D.J. Inman

Three major tasks have been accomplished during the three year funding cycle of the contract. The first was the discovery of instabilities arising in the response of closed loop systems if actuator dynamics are improperly modeled or left out. This constitutes a control/structure interaction (CSI) result. The second accomplished objective was the establishment of an effective method for experimentally verifying and using the inverse procedure discussed under Banks above. Specifically a laboratory was established for the experimental determination of damping mechanisms of composite beams and other structural elements. The third result was the successful derivation of a continuum model for the active vibration suppression of a beam with multilayered piezoelectric actuator. This model was used to perform a preliminary active control study of a fully distributed parameter actuator/structure/sensor system. Each of these three accomplishments is summarized in the following paragraphs. References of papers published as a result of this research are listed following these summaries.

Control/Structure Interaction: Actuators that can be successfully modeled as lumped parameter second order systems were examined for use in vibration control of structures modeled by partial differential equations. A finite dimensional approximation was used to provide insight into the nature of the interaction between control actuator and structural dynamics. This was extended to the distributed parameter case. It was shown that ignoring the actuator dynamics can lead to an unstable closed loop response. The feedback paths available for output feedback control were identified and examined in terms of closed loop stability resulting in closed loop stability conditions for computing stable control gains. An example was developed illustrating that the use of noncolocated feedback results in better closed loop performance than the inherently stable collocated feedback law. An infinite dimensional formulation of a cantilever beam with actuator dynamics has been developed and experimentally verified. The model includes actuator dynamics as well as both internal and external damping mechanisms as modeled by Banks, etal. It remains to complete the computational studies of the infinite dimensional model and its various approximations. Initial experimental verification shows excellent agreement between theoretically predicted and experimentally estimated natural frequencies.

<u>**Experimental Determination of Damping:**</u> A laboratory facility and technique was developed for experimentally verifying and applying the spline based inverse procedures developed under this contract. These procedures incorporate state of the art equipment and methodology for vibration testing. Several samples of proto-type composite beams were used as test articles. The experimental results show clearly that standard modal analysis methods, common to engineering practice, are not capable of measuring and determining the desired distributed parameter coefficients. Several hundred experiments were performed on a variety of test configurations including those with tip bodies. Both time and frequency histories were recorded for responses and inputs. Acceleration histories obtained from traditional accelerometers were compared with more precise measurements made using a laser vibrometer to improve measurement accuracy. Experimental work continues to be performed and is now focusing on measurements of angular acceleration so that more sophisticated continuum models may be approached. In addition, parallel experiments are underway at Edwards Air Force Base in the Astronautics Laboratory's thermo-vacuum facility. This data will eventually allow the determination of temperature effects and the exclusion of air damping in the continuum models.

<u>**Distributed Parameter Actuator/Sensors:**</u> A continuum model of a distributed parameter actuator/sensor system for vibration suppression of flexible members using piezoelectric devices has been derived. The approach taken was to apply Timoshenko theory to beams with multiple layers of piezoelectric material attached. The model is developed using a Hamiltonian approach and includes the external electric circuits as well as a complete set of boundary conditions. The piezoelectric material is segmented and resistors were added to the layers to provide passive damping. A state space model was developed, discretized (via Galerkin methods) and simulated. The finite dimensional approximation was then used to perform open loop and closed loop studies. Velocity feedback was used to examine closed loop control.

This work provides a fully distributed actuator/sensor system for passive and/or active vibration suppression in flexible structures such as airplane wings or space structures. Large increases in damping are obtained by this method without the use of moving parts common to electromechanical actuators.

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Research Summary: K. Ito

I. Control and Approximation of Delay Systems

(1) A regularity property of the solution to an operator Riccati equation arising in the linear quadratic regulator problem is obtained in [1]. The result is used to obtain a rate of convergence of the optimal feedback gain operator in [2].

(2) A Laguerre polynomial based approximation method is developed for a class of infinite delay equations of neutral type, employing the tau and δ−impulse type approximation [3] (with F. Kappel, U. of Graz).

(3) In [4] (with J.A. Burns and G. Propst), we analyze the Galerkin approximation based on linear spline element and show that adjoints of the approximating semigroups converge only weakly (not strongly) to the adjoint semigroup. This fact explains an earlier calculation of Riccati solutions, using the Galerkin approximation which exhibits a zig-zag behavior.

(4) A new spline based approximation [5] (with F. Kappel) is developed. It yields the approximating semigroups that preserve the logarithmic sectorial property (i.e. the differentiability of semigroup) of the solution semigroup. Using such a property, the optimal rate (2nd order) of convergence of the proposed method is established.

(5) A fully-discrete spectral method [6] (with H.T. Tran and A. Manitius) based on Legrendre-polynomials for a class of delay differential equations is developed using the tau approximation in the time variable. A comparison with existing numerical methods has been made. It has been observed that the proposed method not only yields very accurate solution but also is very efficient.

II. Semigroup Approach in Distributed Parameter Systems

(1) A unified formulation of linear elastodynamics with dissipations using sesquilinear forms on Hilbert spaces is developed in [7] (with H.T. Banks). In light of such a formulation, convergence of Galerkin approximation for a wide class of the model equation of dynamics of flexible structure, is established, using the Trotter-Kato theorem.

(2) Motivated by the equation appearing in the modeling of certain aeroelastic system, we study a class of integro-differential equation with singular kernel. In [8] (with J.A. Burns) the equation is formulated as an evolution equation on the weighted L_2−space and a well posedness of its solutions is established. Based on this state-space formulation, in [9] (with J. Turi) numerical methods employing the nonconformal finite element are developed. Convergence of semi- and fully-discrete numerical schemes is established using the semigroup theoretic framework.

(3) In joint work with R. Fabiano, we have studied the linear viscoelastic systems of Boltzman type. In [10], the well-posedness of solution for the case when the kernel is weakly singular is obtained, and we develop and analyze a numerical method for computing solutions to such a system. In [11] the uniform exponential stability and differentiability of the solution semigroup are studied and also a well-posedness result for systems with strong singular kernel is obtained.

III. Inverse Problems

In joint work [12] with Karl Kunisch (Technical University of Graz) we have studied the problem of estimating function parameters in elliptic equations. The problem is formulated as a constrained minimization in Hilbert spaces. The regularization method of Tichonof using semi-norms is used and the behavior of solutions with respect to the regularization parameter is analyzed. A hybrid method that combines the output least squares and equation error methods is developed based on (so-called) augmented Lagrangian method. In [13], a convergence result of the augmented Lagrangian method is established for a general class of nonlinear programming problems in Hilbert spaces. In [14] a use of the augmented Lagrangian method for variational inequalities is discussed, in which the infinite dimensional affine inequality constraints is augmented directly.

Implementations and numerical aspects of the augmented Lagrangian method are discussed in [15]. It contains not only numerical test examples to demonstrate the feasibility of the method but also provides a new convergence theory of the method. In [16], sensitivity analysis of solutions to optimization problems in Hilbert spaces is made. An abstract framework for establishing the Lipschitz continuity and directional differentiability of solution and Lagrange multipliers to parameterized nonlinear programming in Hilbert spaces is developed. We have applied it to problems in optimal control and parameter estimation. A subsequent manuscript will discuss a use of sensitivity analysis for determining the optimal Tichonof -regularization parameter in regularized nonlinear least squares problems in Hilbert spaces. We have also begun to study problem of maximizing the sensitivity of parameter -to-output map F in parameter estimation problems. The smallest singular value is used as a performance index for maximization.

IV. Optimal Control Problems

(1) In [17], a existence result of finite dimensional stabilizing compensator for a class of linear systems in infinite dimensional space is established. Computational methods for constructing such a compensator are developed and their convergence property is analyzed.

(2) Convergence and convergence rate of approximating solutions to operator Riccati equation are obtained under the condition of uniform stabilizability and detectability [2].

(3) In joint work with H.T. Banks, a manuscript on "A Variational Approach to A Class

of Boundary Control Problems" [18] is nearly completed. In this manuscript the linear quadratic optimal control problem for parabolic systems is studied and the boundary control problem is formulated within the framework of Gelfand-triple. Convergence of Galerkin solutions to Riccati equations is established using functional analytic method (especially the theory of analytic semigroups).

(4) Motivated by the above, we (with H.T. Tran) have studied the linear quadratic optimal control problem for a general class of systems involving unbounded input and output operators, especially including the delay differential system [19]. An approximation theory is developed for the solution to operator Riccati equation for the so-called Pritchard-Salamon class. In [20], it is shown that parabolic systems with delays in the control can be treated by the theory developed in [19].

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Research Summary: W.H. Fleming

During the period 9/15/86-9/15/89 Fleming's research focused on stochastic control, non-linear filtering, and related topics in probability and partial differential equations. This work is part of an ongoing study of optimal control of Markov stochastic processes, including Markov diffusions with dynamics governed by stochastic differential equations. There are significant mathematical connections with the theory of viscosity solutions to partial differential equations and partial differential inequalities, and the asymptotic problems for nearly deterministic Markov diffusions (large deviations theory). New kinds of stochastic control problems have arisen from such applications an manufacturing systems and investment-consumption problems. In addition, increasing attention has been given to methods of approximate solution to problems of nonlinear filtering and optimal stochastic control.

Reference [3] gives an overview of research opportunities and promising trends in the field of control as a whole. The various papers in [14] cover a spectrum of areas in stochastic control, including such newer topics as routing and flow control suggested by newer technologies. The lecture notes [15] gives a concise introduction to viscosity solutions in a stochastic control setting.

Control representations for general classes of nonlinear secord-order partial differential equations, of elliptic or parabolic type, were given in [5]. This required a new theory of two-controller, zero-sum stochastic differential games. The approach taken in [7] and [9] was different. It used duality principles from convex analysis, and characterized the solution of the dual optimization problem in terms of maximal smooth subsolutions to the dynamic programming equation.

For special classes of stochastic control problems which have a linear-convex structure, much more information can be obtained. The production planning problem in [4] as well as many investment-consumption problems have such structure. For particular types of one-dimensional problems, the analytical information of [11], based in a viscosity solution method, where combined with the numerical technique of [12], give essentially complete solutions.

In [1] [2] [6] stochastic control and viscosity solution methods were used to study various asymptotic properties of nearly deterministic Markov processes. Included are large deviations properties as well as sharper results in the form of asymptotic series expansions.

Reference [8] introduced a new technique for finding finite dimensional approximations to optimal nonlinear filters. The novel feature is that a many-to-one function of the state (rather than a one-to-one function) is observed with low intensity noise. This work was continued in [10], where results of numerical experiments are reported.

Graduate Students, Postdocs, Visitors

Five students completed the PhD under Fleming's supervision during this period: R. McGwier (1987), D. Ji (1987), Q. Zhang (1988), T. Zariphopoulou (1988), T. Mikami (1989).

Senior visitors. H. Ishii (Chou University, Tokyo) visited for a year. Two senior visitors for periods of from two months to one semester were: R. Ellis (University of Massachusetts, Amherst) and J-L Menaldi (Wayne State University). P-L Lions (Université de Paris, Dauphine) visited for one month. A number of other visitors came for shorter periods, including participants in a workshop in August 1987 on Stochastic Control and Viscosity Solution Methods in Partial Differential Equations.

Postdocs for a period of one semester to one year: G. Barles (visiting from Université de Paris, Dauphine), F. Comets (visiting from Orsay), F. Gozzi (visiting from Scuola Normale Superiore Pisa), B. Fitzpatrick (currently at the University of Tennessee), T. Zariphopoulou (currently at Worcester Polytechnic Institute).

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Abstracts of Research Publications

1. W.H. Fleming and P.E. Souganidis, PDE–viscosity solution approach to some problems of large deviations

   In this paper a new proof is given for result of Fleming and Tsai (Applied Math. Optimiz. vol. 7 (1981) concerning optimal exit probabilities, and the result is extended. The proof is based on viscosity solution techniques for nonlinear partial differential equations. The value of certain differential games is a key quantity in the asymptotic formula obtained for the optimal exit probability. These result give in particular a new proof of the Wentzell–Freidlin results about exit probabilities for nearly deterministic markov diffusions.

2. W.H. Fleming and P.E. Souganidis, Asymptotic series and the method of vanishing viscosity

   We consider second order elliptic quasilinear differential equations (PDEs) depending on a small parameter $\varepsilon > 0$, such that for $\varepsilon = 0$ the equation reduces to first order. The goal is to obtain an asymptotic series in powers of ε for the solution to the Dirichlet boundary problem. Such series are valid in regions where the unperturbed ($\varepsilon = 0$) solution is smooth.

3. W.H. Fleming, S.-J. Sheu and H.M. Soner, A remark on the large deviations of an ergodic Markov process.

   We give an alternative proof of the Donsker–Varadhan result, for the large time behavior of the occupation measure μ_t of a Feller–Markov process $w(t)$, taking values in a compact metric space X. For any weak * continuous function $\phi(\mu)$ on the space of probability measures on X, we identify $\lim_{t\to\infty} t^{-1} ln E \exp(\phi(\mu_t))$ through a linearization argument.

4. W.H. Fleming, S.P. Sethi and H.M. Soner, An optimal stochastic production planning problem with randomly fluctuating demand.

   This paper considers an infinite horizon stochastic production planning problem with demand assumed to be a continuous-time Markov chain. The problems with control (production) and state (inventory) constraints are treated. It is shown that a unique optimal feedback solution exists, after first showing that convex viscosity solutions to the associated dynamic programming equation are continuously differentiable..

5. W.H. Fleming and P.E. Souganidis, Value functions for two-player, zero-sum stochastic differential games

   In this paper we investigate the question of the existence of the value of zero-sum, two-player stochastic differential games. We formulate a notion of value functions which satisfy the dynamic programming principle and are the unique viscosity solutions of the associated Bellman-Isaac partial differential equations.

6. W.H. Fleming and H.M. Soner, Asymptotic expansions for Markov processes with Lévy generators.

   This paper considers a deterministic flow in n-dimensional space, perturbed by a Markov jump process with small variance. Asymptotic expansions are obtained for certain functionals of Feynman–Kac type, in powers of a small parameter representing a noise intensity. The methods are analytical rather than probabilistic.

7. W.H. Fleming, Generalized solutions and convex duality in optimal control.

   The standard Pontryagin of deterministic control theory is reformulated in a "generalized" setting in which a convex duality principle can be applied. The results are related to work of Vinter and Lewis, and earlier ideas of L.C. Young. However, the proofs given are much simpler. The technique is extended to controlled piecewise deterministic processes, in which the state dynamics depend on parameters varying randomly according to a Markov chain.

8. W.H. Fleming and E. Pardoux, Piecewise monotone filtering with small observation noise.

   Nonlinear filtering of Markov diffusion processes is considered, in the case in which a piecewise monotone function of the state is observed with additive small observation noise. Under a certain detectability hypothesis, statistical tests are given to discriminate among the intervals of monotonicity during time intervals in which the state does not cross critical points of the observation function. During such time intervals, accurate approximate finite dimensional filters can be used.

9. W.H. Fleming and D. Vermes, Convex duality approach to the optimal control of diffusions.

   An alternative to the usual dynamic programming approach to the optimal control of markov processes is considered. It is based on duality of convex analysis. The control problem is embedded in a convex mathematical programming problem on a space of measure. The dual problem is to find the supreme of smooth subsolutions to the Hamilton–Jacobi–Bellman equation.

10. W.H. Fleming, D. Ji, P. Salame and Q. Zhang, Piecewise monotone filtering in discrete time with small observation noise.

    A discrete time model for filtering with small observation noise is considered in this paper. A piecewise linear observation function is considered, with two intervals of monotonicity. A sequential quadratic variation test is found to detect intervals of linearity of the observation function. Diffusion approximations to certain discrete processes are made to estimate the means times for reaching a decision and the error probabilities.

11. W.H. Fleming and T. Zariphopoulou. An optimal investment/consumption model with borrowing

    This paper considers a consumption and investment decision problem for a single agent. We treat the problem by the usual method of dynamic programming, which leads to a fully nonlinear second order differential equation for the maximum expected discounted utility $v(x)$ as a function of the investors wealth x. Using viscosity solution methods, sharp asymptotic estimates for $v(x)$ and for optimal control policies are found for small x and for large x.

12. B. Fitzpatrick and W.H. Fleming, Numerical Methods for an Optimal Investment-Consumption Model.

    In this paper, we consider some numerical techniques for determining optimal consumption and investment strategies, based on a discounted utility of consumption cost functional. The discretization method is basically the Markov chain approach of Kushner, but by using viscosity arguments, we are able to prove convergence of the approximate control policies to the optimal policies of the continuous problem.

Research Summary: R.H. Fabiano

During the time period 9/15/86 to 9/15/89 Fabiano collaborated with H.T. Banks, Y. Wang, D.J. Inman and H. Cudney, in efforts which focused on the investigation of models for flexible structures with internal damping and on the development of efficient computational methods for the estimation of parameters in these models. Models were considered with several different types of damping, including external viscous damping, Kelvin-Voigt type viscoelastic damping, time hysteresis and spatial -hysteresis damping, and most recently a model with bending rate damping. The relevant issues of well-posedness and convergence which provide the theoretical foundation for our computational methods were addressed. Our contributions in this area include the development of a well-posed parameter estimation framework and a convergent approximation scheme for an Euler-Bernoulli beam with tip mass and time-hysteresis damping. In addition, numerical packages were tested on data collected from an experimental test structure (a long slender cantilevered composite beam), and favorable results were obtained for the problem of estimating the parameter for the structure.

In work with J.A. Burns, Fabiano considered simulation and control problems for a viscoelastic beam. The work included the development of finite dimensional approximations to infinite dimensional optimal control problems, and these ideas were used to develop algorithms for the computation of approximating feedback gains for these infinite dimensional problems.

In work with K. Ito, Fabiano considered questions of a more theoretical nature for the particular model of Boltzmann -type viscoelastic damping. Well-posedness results for "strongly singular" history kernels were obtained and properties of the solution semigroup for both "strongly" and weakly" singular kernels were investigated. In addition, an approximation scheme, based on a non-uniform mesh in the history variable which gives a significant improvement in convergence rate over other schemes were developed. The scheme has proven to be quite useful for computationally intensive applications in parameter estimation and optimal control for these Boltzmann-type viscoelastic models.

Publications with abstracts

[1] Feedback control of hyperbolic partial differential equation with viscoelastic damping, LCDS/CCS Report, February 1988, Control-Theory & Adv. Tech.., Vol. 5, No.1, March 1989, (with J.A. Burns).

Abstract:In this paper, we consider an approximation scheme be for an optimal control problem described by a hyperbolic partial-functional differential equation used to model the elastic motion of a viscoelastic body of Boltzmann-type. The method is based on combined finite element/averaging approximations. We present theoretical and numerical results for a problem with quadratic cost functional.

[2] Semigroup theory and numerical approximation for equations in linear viscoelasticity, LCDS/CCS Report, July 1988, SIAM J. Math. Anal., to appear, (with K. Ito).

Abstract: We consider the following abstract integro-differential equation

$$\ddot{u}(t) + A[Eu(t) - \int_{-r}^{0} g(s)u(t+s)ds] = f(t)$$

on a Hilbert space. Such equations arise in the modeling of linear viscoelastic beams. The equation is reformulated as an abstract Cauchy problem, and several approximation schemes are discussed. Well-posedness and convergence results are given in the context of linear semigroup theory. Results of numerical eigenvalues calculations for various approximation schemes are discussed.

[3] Inverse problem techniques for beams with tip bodies and time hysteresis damping, ICASE Report No. 89-22; Mat. Aplic. Comput., to appear., (with H.T. Banks and Y. Wang).

Abstract: We present a model for a flexible beam with time hysteresis (Boltzmann-type viscoelastic) damping and tip body. A computational method for the estimation of the damping parameters is developed, and a theoretical convergence/continuous dependence results are given. An example is presented in which experimental data is used, demonstrating the efficacy of the computational method and the plausibility of the model for predicting response in damped structures.

[4] Semigroup theory in linear viscoelasticity: weakly and strongly singular kernels, Proc. Intl. Conf. on Control of Distributed Parameter Systems, Vorau, Austria, July, 1988, submitted, (with K. Ito).

Abstract: We discuss an integro-partial differential equation which arises in the theory of linear viscoelasticity. Previous well-posedness results are extended to include the case of strongly singular kernels. In addition, for the case of a weakly singular kernel and a finite delay, we show that the associated solution semigroup is differentiable.

[5] Spatial versus time hysteresis in damping mechanisms, Proc. 27th IEEE Conf. on Dec. and Control, Austin, TX, December, 1988, (with H.T. Banks, D.J. Inman, H. Cudney, Jr., and Y. Wang).

Abstract: In this report we describe our continuing investigations on the task of estimation internal damping mechanisms in flexible structures. Specifically, we consider two models for internal damping in Euler-Bernoulli beams - spatial hysteresis and time hysteresis. A theoretically sound computational algorithm for estimation is described and experimental results are discussed.

[6] Bending rate damping in elastic systems, Proc. 28th IEEE Conf. on Dec. and Control, Tampa, FL, December, 1989, (with H.T. Banks and Y. Wang).

Abstract: We present some preliminary results from our investigation of bending rate damping model for elastic structures. This is a part of our ongoing effort on the general problem of modeling and parameter estimation for internal damping mechanisms in flexible structures.

Research Summary: H.T. Tran

The research activities of H.T. Tran can be summarized in three main topical areas:

I. *Feedback Synthesis for a Class of Linear Infinite Dimensional Systems* (with K. Ito): Tran and Ito developed a general convergence framework for approximation ideas which can be used in computational techniques of Riccati operators for a class of infinite dimensional systems with unbounded input and output operators. The convergence theory was developed in the context of the theory of infinite dimensional Riccati equations pursued recently by Pritchard and Salamon. The novel feature of their work (and hence of the Tran/Ito work) is that: (i) it allows for unbounded input and output operators, i.e., boundary control, point observation, input and output delays, and (ii) it applies to both parabolic and hyperbolic partial differential equations as well as retarded and neutral functional differential equations. For applications, Ito and Tran showed how the abstract approximation framework can be used for a numerical scheme based on the averaging approximation method to solve the infinite dimensional Riccati equations in connection with hereditary control problems involving point delays in the inputs. They also showed that a class of parabolic partial differential equations with delays in control fitted the abstract framework of Pritchard and Salamon and thus the related Riccati operator and approximation theory.

II. Modeling and Parameter Estimation Problems for Biological Systems (with H.T. Banks and D.E. Woodward): Tran, in collaboration with Banks and Woodward developed a convergence theory for a computational techniques combining a variation of the moving finite element method with spline approximation for the problem of estimating variable (temporally and spatially dependent) coefficients in the Fokker Planck or forward Kolmogorov equations. These models, based on a Markov transition assumption for the growth process describe the evolution in time in populations in which individual size or age plays an important role. The computational scheme developed allows one to treat estimation problems in Fokker Plank systems where the drift or convective term dominates the diffusion. These systems are difficult even for simulation studies since traditional finite difference or finite element method gives invalid solutions. In another joint effort with K. Ito, S. Michelson (Syntex Corporation, IRDM) and J.T. Leith (Rhode Island Hospital), Tran derived a stochastic model describing the emergence dynamics of one subpopulation from another in heterogeneous tumors. The resulting system is a pair of Fokker Planck equations. These equations were numerically solved by finite element method to simulate several biological and clinical scenarios and a comparison of these simulations with the dynamics of its deterministic counterpart were also made. The clinical and biological importance of the findings were also carefully discussed.

III. Numerical Approximations of Delay Differential Equations (with K. Ito): Tran and Ito analyzed the averaging approximations for linear retarded functional differential equations with delays in control and observations. It was shown that the approximating semigroups are all uniformly differentiable with respect to the index N determining the mesh size. This fact makes it possible to prove the existence of a uniform exponential decay (or growth) rate of the approximating semigroups and to establish convergence rates of the homogeneous solutions. The development presented in this work gives a basis for numerical investigations of control problems, in particular using the operator Riccati equations. In other research with K. Ito and A. Manitius (School of Information Technology and Engineering, George Mason University), Tran analyzed a fully discrete Legendre-tau method for the numerical solution of linear delay differential equations. The method was shown to provide a reliable and fast integration scheme with global spectral accuracy. Stability analysis of the method in L^2 and H^1 norms also was obtained. An efficient implementation of this scheme requires special integration techniques for the calculation of the Legendre coefficients. In another related work, Tran discussed a reliable, economical and highly accurate method for the numerical calculations of Legendre coefficients. The method consists of expending the Legendre integral exactly in a finite series of Fourier coefficients. He then numerically showed that the Fourier coefficients can be computed with negligible truncation error by the Filon scheme with Richardson extrapolation.

Publications with abstracts.

[1] A numerical method for the integration of rapidly oscillating functions, (with K. Ito), LCDS/CCS Report 88-7, April 1988, Math. Comp., submitted.

Abstract: A method for the numerical calculations of the integrals

$$I(n) = \int_{-1}^{1} f(x)p_n(x)dx,$$

where $p_n(\cdot)$ is a Legendre polynomial of degree n, is presented. The method consists of expanding the integral $I(n)$ exactly in a finite series of Fourier sine in integrals. Therefore, special integration methods developed for evaluating Fourier integrals can then be applied to approximate $I(n)$. Numerical examples demonstrate that this method is very accurate, economical, and reliable.

[2] Numerical approximations for hereditary systems with input and output delays: convergence results and convergence rates, (with A. Manitius), LCDS/CCS Report 88-14, July 1988, SIAM J. Control and Opt., submitted.

Abstract: In this paper, the averaging approximation scheme for linear retarded functional differential equations with delays in control and observation are considered in the context of the state space theory developed by Pritchard and Salamon. Using known results from linear semigroup theory, convergence and estimate of convergence rate of the approximating semigroups are established. These are extended results due to Banks and Burns, and Lasiecka and Manitius on hereditary systems with delays in state to the vase when delays in control and observation are included. The main difference from the case when delays in input and output are

excluded is that we have to deal with an unbounded input and output operators in our abstract formulation. Moreover, in the presence of unboundedness of the input and output operators, new convergence results of the state solutions and the output are also obtained.

[3] Stochastic models of subpopulation emergence in heterogeneous tumors, (with S. Michelson, K. Ito and J.T. Leith), Proc. 12th IMACS World Congress on Scientific Computation, Paris, France, July 1988.

Abstract: A system of Fokker-Planck equations describing the probability density function for the size of two species in a random environment is derived from a variant of the general Lotka-Volterra model for interspecific competition. The variant described the emergence of one subpopulation from another as a constant mutation rate. While note relevant to the macro-ecological model describing animal interactions, the variant can be used to model heterogeneous tumor growth. Numerical solutions to the partial differential equation system suggest that even though the mean trajectory described by the stochastic model and the primary trajectory described by the deterministic systems coincide, measurable extinction probabilities exist for trajectories that approach the axes. The biological and clinical implications for these results is discussed.

[4] Stochastic models for subpopulation emergence in heterogeneous tumors, (with S. Michelson, K. Ito and J.T. Leith), LCDS/CCS Report 88-18, July 1988, Bul. of Math. Biol., to appear.

Abstract: A stochastic analog to a deterministic model describing subpopulation emergence in heterogeneous tumors is developed. The resulting system is a pair of Fokker-Planck equations. A finite element approach for the numerical solution to these equations is described. Four biological and clinical scenarios are simulated (emergence of heterogeneity, exclusion of a subpopulation, and induction of drug resistance in both pure and heterogeneous tumors). The results of the simulations

show that the stochastic model described the same basic dynamics as its deterministic counterpart via a convective component, but that for each simulation a distribution of tumor sizes and mixes can also be derived from a diffusive component in the model. These distributions yield estimates for subpopulation extinction probabilities. The biological and clinical revalence of these results are discussed.

[5] Linear quadratic regulator problems for infinite dimensional linear systems with delays in control, (with K. Ito), LCDS/CCS Report 88-32, September 1988, Proc. 27th IEEE Conf. on Dec. and Control, Austin, TX, December, 1988.

Abstract: We consider the linear quadratic optimal problem for linear control systems with delays in control defined on Hilbert space H_0 and develop numerical approximation method for approximate Riccati operators. We show how to reformulate the problem as a boundary control problem using the product space $H_0 \times L_2$ as the state space. We then present an approximation framework for computing the Riccati operator (in finite dimensional spaces) that can be guaranteed to converge to the Riccati operators of the infinite dimensional systems with unbounded input.

[6] Linear quadratic optimal control problem for linear systems with unbounded input and output operators: numerical approximations, (with K. Ito), LCDS/CCS Report 88-33, November 1988, Proc. Intl. Conf. on Control of Distr. Para, Sys., Vorau, Austria, July 1988, to appear.

Abstract: We present an abstract approximation framework for the numerical treatment of Riccati operators for a class of linear infinite dimensional systems with unbounded input and output operators. As a simple application, we show how the results of the general convergence theory may be applied and hence solve the linear quadratic control problem for linear functional differential equations with point delays in the controls.

[7] A fully-discrete spectral method for delay-differential equations, (with K. Ito and A. Manitius), LCDS/CCS Report 88-34, November 1988, SIAM J. Numer. Anal., submitted.

Abstract: In this paper we introduce a new Lanczos-tau method for solving linear functional differential equations. The scheme has infinite order of accuracy both in time and in the delayed argument. The high accuracy in time is obtained without increasing the computational work and memory space which is needed for a one step explicit difference scheme. We demonstrate how to implement the algorithm in a robust and efficient manner and to treat problems with piecewise continuous initial function. Numerical results illustrating the behavior of the method when faced with difficult problems are presented and a comparison with other methods published in the literature is made.

[8] Estimation of variable coefficients in the Fokker Planck equations using moving finite elements, (with H.T. Banks and D.E. Woodward), in preparation.

Abstract: We consider the inverse problems for the estimating of temporally and spatially varying coefficients in the Fokker Planck or forward Kolmogorov equations that often arise in size/age structured population modes. These are difficult problems even for simulation studies since they share certain numerical difficulties with transport dominated diffusion-convection problems of fluid dynamics. These difficulties stem from the fact that when the convection is large compared to the diffusion, both traditional finite difference or finite element methods produce erroneous oscillatory solutions. In this paper we present computational techniques combining a variation of the moving finite element method with spline approximations for the parameter estimation problems. Computational details of our numerical algorithm along with theoretical convergence results are presented. Several numerical examples illustrating the effectiveness of the method are also given.

Research Summary: B.G. Fitzpatrick

Research of B.G. Fitzpatrick at the Center for Control Sciences focused on three major themes: statistical techniques for the analysis of least squares estimators for parameter identification problem, modeling and estimation in size structured populations, and numerical methods for stochastic control problems. These efforts may be abstracted as follows:

Statistical Theory of Least Squares. In many applications of least squares parameter identification, the observations one wishes to fit have been corrupted by a noise process. The research in this area examines some of the statistical properties of least squares estimators. The goals of such a study were twofold: firstly, one wishes to know if the estimator is "close" to the "true" parameter value, and secondly, one wishes to know if a reduction in the parameter space might yield an estimator which is "just as good" as the estimator obtained from the original parameter space.

Consider the collection of observations

$$Y_k = f(x_k, q^*) + \epsilon_k, 1 \leq k \leq n.$$

where $\{Y_k\}$ is the collection of observation, $\{x_k\}$ is a collection of settings at which measurements are made ($x_k \in X \subset \mathbf{R}^m$), and $\{\epsilon_k\}$ is the noise process. The parameterized function $f(x, q)$ is a model to which one wishes to fit the observations. Usually, the function f arises from a parameter-dependent differential equation which is derived from physical considerations of the system studied.

The first question is that of estimation of the parameter q. The least squares cost functional used is

$$J_n(q) = \frac{1}{n} \sum_{k=1}^{n} (Y_k - f(x_k, q))^2, \quad q \in Q_{ad}.$$

Let $\hat{q}_n$ denote a minimizer of J_n. Fitzpatrick and Banks have proved that $\hat{q}_n$ converges to the "true" parameter q^*; that is, the parameter that generated the data.

This result has been extended to cases in which f must be approximated and the parameter space must be approximated. The approximation must be taken into account because in general only approximations of f can be computed, and in many cases the parameters are actually functions.

Goodness of fit questions are much more difficult. They involve a smaller set of parameters, Q_0 is which one thinks the true parameter may lie. An ANOVA-type test was used to examine these questions. To perform the hypothesis test $H_0 : q^* \in Q_0$, versus $H_1 : q^* \notin Q_0$, asymptotic distributions for the ANOVA test statistic were derived, which allows one to compute rejection regions. The techniques have been applied to convection questions in biological data, as well as damping models in viscoelastic beams.

Modeling and Estimation in Size Structured Populations. The Sinko-Streifer model for size structured population evolution is given by

$$v_t + (gv)_x = -\mu v$$

where $v(x,t)$ denotes the density of individuals of size x at time t, g denotes the individual growth rate, and μ denotes the mortality rate. Unfortunately this model does not predict certain behavior (in particular, development of two modes and dispersion is sizes) that has been observed experimentally in some populations. To devise a model which captures the phenomena, Fitzpatrick and Banks treat the population as made up of individuals with growth rates which are independent, identically distributed random functions, rather than individuals with identical growth rates. This modeling assumption leads to a population density of the form

$$u(t,x) = \int_G v(t,x;g)P(dg),$$

where P is the distribution of the growth rate, G is the collection of all possible growth rates, and v is the (g dependent) solution of the Sinko-Streifer equation. The problem of interest to biologist, then, is to estimate P from an observed population density. An approximation framework was developed which allows one to use convex combinations of point masses to match the mouel to the data via least squares minimization. The convergence theory tells one that this minimizer must be close to the underlying P, for a large enough collection of point masses.

Numerical Methods in Stochastic Control. The research (Fitzpatrick and Fleming) in this area was initiated with an investment-consumption model. Markov chain approximation techniques were applied to find a suitable discretized problem, which was implemented in a computer program. Using viscosity solution arguments, it was found that not only do the discrete value functions converge, but also that the discrete optimal policies converge.

Publications with abstracts.

[1] Inverse problems for distributed systems: statistical tests and ANOVA, (with H.T. Banks, in Proc. Intl. Symp. in Mathematical Approaches to Environ. and Ecol. Problems, Springer Lecture Notes in Biomathematics, to appear.

Abstract: In this note we outline some recent results on the development of a statistical testing methodology for inverse problems involving partial differential equation models. Applications to biology and mechanics are presented. The statistical tests are based on asymptotic distributional results for estimators and residuals in a least squares approach.

[2] Statistical tests for model comparison in parameter estimation problems for distributed parameter systems, (with H.T. Banks), CAMS Report 89-4, Center for Applied Mathematical Sciences, University of Southern California, Los Angeles, CA 90089, 1989, and accepted for publication by *The Journal of Mathematical Biology.*

Abstract: We derive a statistical theory of least squares estimation in inverse problems. The results, which are applied in the spirit of ANOVA, are asymptotic theorems for the estimators and residual.

[3] Estimation of growth rate distribution in size structured population modeling, (with H.T. Banks), in preparation.

Abstract: In this work, we develop a new model for size structured populations. The new model, based on the Sinko-Streifer equation, involves a distribution of individual growth rates, and captures more of the observed behavior of some populations of interest. We also develop an approximation framework for using least squares estimation to compute the growth rate distribution from observed population data.

[4] Numerical methods for an optimal investment-consumption model, (with W.H. Fleming), LCDS/CCS Report 88-16, Division of Applied Mathematics, Brown University, Providence, RI.

Abstract: We consider a Markov chain discretization of an investment-consumption model from financial economics. Results of convergence of the discrete value function and the discrete control policies are derived. Numerical examples illustrate not only this convergence, but also interesting policies for non-HARA utility functions.

Research Summary: Karl Kunisch

Karl Kunisch's work associated with the AFOSR-URI grant was carried out in collaboration with Professor H.T. Banks and Professor K. Ito.

1) Joint work with Professor Banks.

The collaboration with Professor H.T. Banks concentrated on parameter estimation problems. Different approximation techniques for nonlinear least squares problems for the identification of parameters in partial differential equations were developed and a framework for convergence proofs was established. This work lead to several Ph.D. and Master Theses, and together with a wide variety of real world applications it is presented in the form of a monograph [1].

2) Joint work with Professor Ito.

The work concentrated on abstract smooth optimization problems and on parameter estimation problems. The augmented Lagrangian technique for equality and inequality constraints in Hilbert spaces was investigated, and convergence as well as rate of convergence results were obtained without strict complementarily assumption [2]. These results were applied to variational inequalities [3] and to parameter estimation problems for elliptic partial differential equations [4,5]. The augmented Lagrangian formulation for estimation of coefficients in partial differential equations can be interpreted as a hybrid method between the least squares and the equation error techniques. In fact it combines the advantages of both of these methods and proved to be very reliable in many test examples [7].

Further work was carried out for sensitivity analysis of abstract optimization problems. It was shown that a certain second order sufficient optimality condition implies directional differentiability of the solution of the optimization problem with respect to the problem data. The problem formulation allows equality as well as inequality constraints and is applicable to optimal control problems with state and control constraints and to parameter estimation problems. These results also provide the theoretic foundation for a new technique to determine the regularization parameter in illposed inverse problems. It is based on a model function technique, with the model function describing approximately the behavior of the regularized cost functional as a function of the regularization parameter. The model function itself is a solution of a second order ordinary differential equation, which is obtained by taking appropriate derivatives of the cost functional and of the regularized solution with respect to the regularization parameter. A preprint of a major part of this work is already available [8].

A further project under investigation is the determination of optimal input functions for parameter estimation problems.

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[1] H.T. Banks and K. Kunisch: "Estimation Techniques for Distributed Parameter Systems," Birkhäuser, Boston, 1989.

[2] K. Ito and K. Kunisch: The augmented Lagrangian method for equality and inequality constraints in Hilbert spaces, to appear in Mathematical Programming.

[3] K. Ito and K. Kunisch: An augmented Lagrangian technique for variational inequalities, to appear in Appl. Math. and Opt.

[4] K. Ito and K. Kunisch: A hybrid method combining the output least squares and the equation error approach for the estimation of parameters in elliptic systems, to appear in SIAM J. on Control and Opt.

[5] K. Ito and K. Kunisch: The augmented Lagrangian method for estimating the diffusion coefficient in an elliptic equation, in Proc. 26th IEEE Conf. on Dec. and Control, Los Angeles 1987, 1400-1404.

[6] K. Kunisch: Contributions to nonlinear inverse problems arising in parameter estimation for elliptic systems, to appear in: Proc. Conf on Future Trends in Control Theory, Sielpia, Poland, 1988.

[7] K. Ito, M. Kroller and K. Kunisch: A numerical study of the augmented Lagrangian method for the estimation of parameters in elliptic systems, submitted to SIAM J. on Sci. and Stat. Computing.

[8] K. Ito and K. Kunisch: On the choice of the regularization parameter in nonlinear inverse problems, preprint.

[9] K. Kunisch and L. White: Stability and identifiability of parameters in Galerkin approximations of distributed systems under multiple forcing, preprint.

Research Summary: D.E. Woodward

For the time period 8/1/88-7/31/89, D.E. Woodward worked on the following projects at the Center for Control Sciences:

1. Numerical solution of partial differential equations. Woodward was involved in the development of a numerical method for the solution of the Fokker Planck or forward Kolmogorov equations for Markov transition growth models. The combination of the convection and diffusion terms, which compete to govern the dispersal behavior of the solution make these difficult problems even for simulation studies. Motivation for this work comes from investigations of two different aquatic populations mosquito fish (Gambusia affinis) in rice fields, and striped bass (Morone saxatilis) in the larval stage before they reach sizes at which recruitment is measured. This was joint work with H.T. Banks and H.T. Tran.

2. Adaptive dynamical systems. Woodward investigated the phase-locking properties of systems on nonlinear coupled oscillators with applications to neutral networks and integrated circuits. This work was primarily concerned with a model for neuron behavior (the VCON) that is posed in terms of electrical circuits that are directly modelled by phase-variables. Numerical simulations show that the potential generated by the VCON strongly resembles the membrane potential recorded in space-clamp experiments on squid giant axon. The arrangement of some parts of the brain in barrel-like structures suggested the study of circular arrays of VCONs. The rotation vector (which is an N-dimensional analog of the rotation number of Poincaré and Denjoy) was used to establish the existence of synchronized states that are stable under persistent disturbances. Related to these ideas is the construction of the Lyapunov (or energy) function for the network, the graph of which can be thought of as a frequency-response surface that is similar to the computational energy of the collective-decision circuits of Hopfield et al.

REFERENCES

[1] H.T. Banks, H.T. Tran and D.E. Woodward, "Estimation of variable coefficients in Fokker Planck equations using moving finite elements", in preparation.

[2] D.E. Woodward, "Synthesis of phase-locking patterns in networks with cyclic group symmetries", SIAM J. Appl. Math., submitted.

Research Summary: C. Dafermos

Dafermos continued his research on the theory of weak solutions of genuinely nonlinear, hyperbolic systems of conservation laws. By using generalized characteristics and entropy estimates, he established sharp rates of decay of solutions with initial data in L^1.

Dafermos and X. Geng studied, by the same techniques, a special system of conservation laws that governs the process of separating three ionized chemical compounds by the electrophoretic method of isotachophoresis. They established uniqueness and generic regularity of solutions.

Research Summary: D. Gottlieb

David Gottlieb and his co-workers have made progress in several areas of numerical solutions of partial differential equations.

1. H. Talezer (a post doc supported by the URI grant) has continued his research in the area of efficient implementation of spectral methods. Talezer explored the possibility of improving the CFL condition for both hyperbolic and parabolic equations that are numerically solved by Spectral methods.

Talezer has proposed a new method to re-distribute the grid points and thus to gain a better stability condition. His method is being extensively checked from the practical as well as the theoretical side. It might be a breakthrough in applying spectral methods to delay equations.

2. David Gottlieb and R. Hirsh have continued their investigation of domain decomposition techniques in spectral methods as a way to apply spectral methods with the use if parallel computers. The interconnection of the individual domains is the stumbling block in fully parallelizing a given method. They have shown a way to minimize the interconnection between the domains for 2D elliptic systems. Mr. Kelly Black is applying this idea on the Intel Hypercube at Brown University with considerable success. He is reporting savings of order of magnitude using the new method.

3. A new effort of applying wavelets instead of trigonometrical polynomials in spectral methods is under investigation. It seems that for problems for flow control, wavelets are more natural that Fourier polynomials. Preliminary results show that wavelets can serve as an alternative to the classical spectral methods.

4. In a joint effort with W.S. Don, the problem of controlling flow behavior via boundary conditions was explored. The problem arises in the numerical simulations of wake flow past a circular cylinder. A recent paper by Sreenivasan showed experimentally that the main shedding frequency is modulated by a lower second frequency. Numerical results by Rudy and Townsand seemed to agree with this observation. Gottlieb and Don showed that this secondary frequency is a numerical artifact caused by choosing the wrong boundary conditions. The right way of imposing boundary conditions in order to let numerical disturbances go out through the boundaries has been utilized successfully. Moreover, it turned out that what was measured in the wind tunnel was effected by the walls of the wind tunnel.

5. In connection with the last item, Gottlieb and Funaro developed a new treatment of boundary conditions for hyperbolic equations, this novel treatment combines imposing the boundary conditions with satisfying the equations themselves at the boundary.